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Non-Darcy free convection induced by a vertical wavy surface in a thermally stratified porous medium

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Abstract

The partial differential equations governing the natural convection heat transfer from a vertical wavy surface in a thermally stratified fluid saturated porous medium are analysed under Forchheimer based non-Darcian assumptions. Based on non-similarity transformation deduced by scale analysis the governing equations are reduced to boundary layer equations. These simplified partial differential equations are solved numerically by a finite difference scheme following the Keller Box approach. Extensive numerical simulations are carried out for various values of wavelength-to-amplitude ratio of wavy vertical surface at different thermal stratification levels of porous medium both under Darcian and non-Darcian assumptions. Results from the current study are compared with those available in literature. In Darcian case local heat fluxes along the wavy vertical surface are periodic with an oscillatory pattern of period, which is exactly half of the period of vertical wavy surface. In the non-Darcian case local heat fluxes continue to be periodic but with a complex oscillatory pattern of period exactly same as that of the vertical wavy surface. Increasing S or Gr^* or a leads to a fall in local Nusselt number.

1. Introduction

Natural convection in a fluid-saturated porous medium is of fundamental importance in many industrial and natural problems. Few examples of the heat transfer by natural convection can be found in geophysics and energy related engineering problems such as natural circulation in geothermal reservoirs, acquifers, porous insulations, solar power collectors, spreading of pollutants etc. Natural convection occurs due to the spatial variations in density, which is caused by the non-uniform distribution of temperature or/and concentration of a dissolved substance. There are many studies [1–5] in which natural convection caused by immersing a hot surface in a fluid-saturated porous medium at constant ambient temperature has been considered. Also a few studies are found when the porous medium is thermally stratified, i.e. the ambient temperature is not uniform and it varies as a linear function of stream wise direction. This phenomenon has its applications in hot dike complexes in volcanic region for heating of ground water, development of advanced technologies for nuclear waste management, separation process in chemical engineering etc. Rees and Lage [6], Takhar and Pop [7] and Tewari and Singh [8] analytically analysed free convection from a vertical plate immersed in a thermally stratified porous medium under boundary layer assumptions. On the other hand Angirasa and Peterson [9], Rathish Kumar and Singh [10] and Rathish Kumar et al. [11] have numerically investigated the natural convection process in a thermally stratified porous medium.

Above mentioned studies of natural convection heat transfer are focused mainly on the flat surface assumptions. However irregular surfaces are often present in many applications and hence it is required to study the effect of such complex geometries on the convection process. Such irregular surfaces are mainly encountered in heat transfer devices to enhance the heat transfer, for

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Nomenclature

$\bar{a}$	dimensional amplitude of the wavy surface
a	non-dimensional amplitude of the wavy surface
f	non-similar stream function
g	acceleration due to gravity
Gr^*	modified Grashof number $(Kg\beta(t_w - t_{\infty,0})K'/v^2)$
K	permeability of the porous medium
K'	coefficient of inertia
ℓ	wavelength of the surface waves
Nu	Nusselt number
Ra	modified Rayleigh number $(Kg\beta\ell(t_w - t_{\infty,0})\alpha/v)$
s	dimensional thermal stratification term
S	non-dimensional thermal stratification term
t	dimensional temperature
T	non-dimensional temperature
$\bar{u}$, $\bar{v}$	dimensional velocity components
x, y	dimensional Cartesian co-ordinates
X, Y	non-dimensional Cartesian co-ordinates
$\prec$	almost less than
$\succ$	almost greater than

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
σ	sinusoidal wavy surface $(\sigma(X) = \frac{a}{\pi}\sin(\pi X - \phi))$
(ξ, η)	transformed co-ordinate variables
$\overline{\Psi}$	dimensional stream function $(u = \partial\overline{\Psi}/\partial y$, $v = -\partial\overline{\Psi}/\partial x)$
v	kinematic viscosity
ϕ	phase of the wavy surface
Ψ	non-dimensional stream function

Subscripts

$0, \infty$	ambient points
w	evaluated at the wall
x	evaluated at point x

example, flat plate solar collectors and flat plate condensers in refrigerators. Bhavnani and Bergles [12] and Yao [13] considered the effect of surface roughness in the context of continuum fluid. Rees and Pop [14,15], Rathish Kumar et al. [16], Rathish Kumar and Shalini [17], Cheng [18] etc. studied such phenomena in fluid-saturated porous medium.

It has been found that Darcy law is valid strictly for low-speed flows or where the Reynolds number based on the volume-averaged velocity and pore length scale is less than O(1) [1,19]. However for a high-speed flow the non-Darcy law is found to be more appropriate. Bejan and Poulikakos [19], Plumb and Huenefeld [20], Kumari et al. [21] etc. have studied the similar flow for the natural convection using non-Darcy model. Rathish Kumar and Shalini [22–24] solved a series of problems to consider the natural convection process in a non-Darcian thermally stratified vertical wavy porous enclosure.

In the present study we aim to consider the natural convection process in a thermally stratified fluid-saturated porous medium under the boundary layer assumption. Non-Darcy assumptions based on the Forchheimer model are invoked in the momentum equations. The full non-linear, coupled momentum and energy equations are simplified by using scale analysis and with the assumption that $Ra \gg 1$. The resulting non-similar boundary layer equations are solved, using an implicit finite difference scheme based on the Keller Box approach [25]. The obtained results are presented in terms of the local and average Nusselt number plots and the stream function and temperature contours.

2. Mathematical formulation

We consider a vertical surface with transverse sinusoidal undulations embedded in an isotropic fluid-saturated porous medium. Flow model and the co-ordinate system is shown in Fig. 1. The origin of the co-ordinate system is placed at the leading edge of the vertical sur-

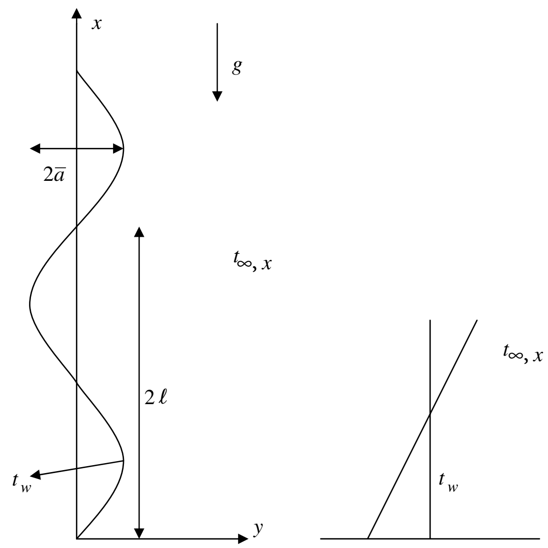

Fig. 1. Schematic diagram of the physical model and the co-ordinate system.

face. The surface profile of the vertical wavy wall is given by

$$y = \sigma(x) = \bar{a}\sin\left(\frac{\pi x}{\ell} - \phi\right) \tag{1}$$

where $\bar{a}$ is the amplitude of the surface wave, 2ℓ is the wavelength and ϕ is the phase of the wave. The wavy surface is considered at constant temperature t_w. Sufficiently far from the vertical wall porous medium temperature is considered as, $t_{\infty,x}$, where $t_{\infty,x} = t_{\infty,0} + sx$, $s = t_{\infty,x}/\mathrm{d}x$, x being the cartesian co-ordinate in vertical direction. Flow is considered to be steady and two-dimensional and assumed to satisfy the Boussinesq approximation.

The non-Darcy momentum equation under the Forchheimer assumptions and the energy equation governing the fluid flow and heat transfer in homogeneous and isotropic porous medium can be written as follows in terms of the non-dimensional variables:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} + Ra^{-1}\,Gr^*\left[\frac{\partial}{\partial Y}\left(\frac{\partial \Psi}{\partial Y}\right)^2 - \frac{\partial}{\partial X}\left(\frac{\partial \Psi}{\partial X}\right)^2\right]$$
$$= Ra\frac{\partial T}{\partial Y} \tag{2}$$

$$\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} = \frac{\partial \Psi}{\partial Y}\frac{\partial \Psi}{\partial X} + S\frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial X}\frac{\partial T}{\partial Y} \tag{3}$$

with the boundary conditions:

$$\Psi = 0, \quad T = 1 - SX \quad \text{on } Y = \sigma(X) = \frac{a}{\pi}\sin(\pi X - \phi)$$
$$\frac{\partial \Psi}{\partial Y} \to 0, \quad T \to 0 \quad \text{as } Y \to \infty \tag{4}$$

The non-dimensional variables are defined as follows:

$$X = \frac{x}{\ell}, \quad Y = \frac{y}{\ell}, \quad a = \frac{\bar{a}\pi}{\ell}, \quad \Psi = \frac{\overline{\Psi}}{\alpha}, \quad T = \frac{t - t_{\infty,x}}{t_w - t_{\infty,0}},$$
$$Ra = \frac{Kg\beta\ell(t_w - t_{\infty,0})}{\alpha v},$$
$$S = \frac{1}{(t_w - t_{\infty,0})}\frac{\mathrm{d}t_{\infty,x}}{\mathrm{d}X} \quad \text{and} \quad Gr^* = \frac{Kg\beta(t_w - t_{\infty,0})K'}{v^2} \tag{5}$$

Here K' is defined as, $K' = C_f K^{1/2}$ where C_f a dimensionless form-drag constant [1]. Other variables have their usual meaning as given in nomenclature.

Effect of the wavy surface is transformed from the boundary conditions to the governing equations by means of the following transformations, obtained by using the usual boundary layer scale analysis

$$\xi = X, \quad Y = \xi^{1/2} Ra^{-1/2}\eta + \sigma(X)$$
$$\Psi = Ra^{1/2}\xi^{1/2} f(\xi, \eta) \tag{6}$$

By substituting the transformation (6) into Eqs. (2)–(4) and in the limiting case $Ra \to \infty$, we obtain the following transformed boundary layer equations

$$(1 + \sigma_\xi^2)\frac{\partial^2 f}{\partial \eta^2} + Gr^*(1 + \sigma_\xi^3)\frac{\partial}{\partial \eta}\left(\frac{\partial f}{\partial \eta}\right)^2 = \frac{\partial T}{\partial \eta} \tag{7}$$

$$(1 + \sigma_\xi^2)\frac{\partial^2 T}{\partial \eta^2} + \frac{1}{2}f\frac{\partial T}{\partial \eta} = \xi\left(\frac{\partial f}{\partial \eta}\frac{\partial T}{\partial \xi} + S\frac{\partial f}{\partial \eta} - \frac{\partial f}{\partial \xi}\frac{\partial T}{\partial \eta}\right) \tag{8}$$

together with the boundary conditions:

$$f = 0, \quad T = 1 - S\xi \quad \text{on } \eta = 0$$
$$\frac{\partial f}{\partial \eta} \to 0, \quad T \to 0 \quad \text{as } \eta \to \infty \tag{9}$$

Local Nusselt number along the vertical wavy surface at a streamwise location ξ is given by

$$\frac{Nu_\xi}{Ra^{1/2}\xi^{1/2}} = -T'(\xi, 0)(1 + \sigma_\xi^2)^{1/2} \tag{10}$$

Average Nusselt number between the leading edge ($\xi = 0$) and a streamwise location $\xi = \overline{X}$ is given by

$$\frac{Nu_m}{Ra^{1/2}} = -\overline{X}\,\frac{\int_0^{\overline{X}} \frac{(1+\sigma_\xi^2)T'(\xi,0)\,\mathrm{d}\xi}{\xi^{1/2}}}{\int_0^{\overline{X}}(1 + \sigma_\xi^2)^{1/2}\,\mathrm{d}\xi} \tag{11}$$

The non-similar boundary layer equations (7)–(9) are solved using the Keller Box approach [25] in finite difference framework upto an accuracy of 10^{-10} on all the variables at each grid point. A uniform step size of 0.05 was used in ξ-direction and a logarithmic grid of 500 points, concentrated towards $\eta = 0$, was used in η-direction. Double precision arithmetic was used throughout and η_∞ was taken as 20, which was found to be well outside the boundary layer.

3. Results and discussion

The parameters which govern the free convection process from a vertical wavy surface to a thermally stratified porous medium under the influence of non-Darcian inertial forces are: (i) thermal stratification level (S), (ii) amplitude-to-wavelength ratio (a), (iii) wave phase (ϕ) and (iv) modified Grashof number (Gr^*). The flow and the temperature distributions are analysed both under Darcian and non-Darcian assumptions in thermally stable and thermally stratified porous medium.

Before carrying out a detailed study, the accuracy of the results generated by the solution methodology is ensured by comparing the same with those from the literature. In Table 1, $T'(0,0)$ and $f'(0,0)$ obtained in the current study under non-Darcian assumptions for flat vertical wall case when $S=0$, are compared with results of Plumb and Huenefeld [20]. For the undulated vertical wall case under Darcian assumptions with the expression proposed by Rees and Pop [15] the current approach reproduces qualitatively and quantitatively the same variation of local Nusselt number (Nu_ξ) as one moves along the wavy wall for $0 \leqslant a \leqslant 0.6$ as reported by [15].

3.1. Influence of thermal stratification (S) on free convection process

To begin with the influence of thermal stratification level (S) on free convection from a vertical undulated surface with $a=0.2$, $\phi=0°$ both under Darcian and non-Darcian assumptions is analysed for $S=0.0$, 0.025, 0.05, 0.075, 0.1. Owning to the fact that in this study $0 \leqslant \xi \leqslant 10$ and $T=1-S\xi$ on the wavy wall, $S=0.1$ corresponds to the case wherein there is a 100% thermal stratification i.e. the non-dimensional temperature T along the wavy wall varies from $T=1$ at $\xi=0$ to $T=0$ at $\xi=10$. In Fig. 2(a) local Nusselt number variations (Nu_ξ) along the wavy wall (i.e. $0 \leqslant \xi \leqslant 10$) under the Darcian and non-Darcian assumptions for various values of S are presented. In both the cases when $S>0.0$, in tune with the boundary condition $T=1-S\xi$, the local heat transfer is seen to gradually decrease as ξ increases. Increasing the thermal stratification level (S) brings in an increased reduction in Nu_ξ all along the wall. When $S=0.1$ one may note that Nu_ξ gets zeroed down at $\xi=10$. For the Darcian case variation of Nu_ξ along the wall has a wavy pattern, indicating a raise and fall in the local heat fluxes. Presence of thermal stratification leads

Table 1
Comparison of the T' and f' values obtained in the present study with Plumb and Huenefeld [20] in non-Darcian case, when $a=0$, $S=0$, $\xi=0$, $\eta=0$

Gr^*	$T'(0,0)$		$f'(0,0)$	
	Plumb and Huenefeld [20]	Present work	Plumb and Huenefeld [20]	Present work
0.0	−0.44390	−0.44374	1.00000	0.99999
0.01	−0.44232	−0.44216	0.99020	0.99019
0.1	−0.42969	−0.42950	0.91608	0.91608
1.0	−0.36617	−0.36575	0.61803	0.61803
10.0	−0.25126	−0.25065	0.27016	0.27016
100.0	−0.15186	−0.15145	0.09512	0.09512

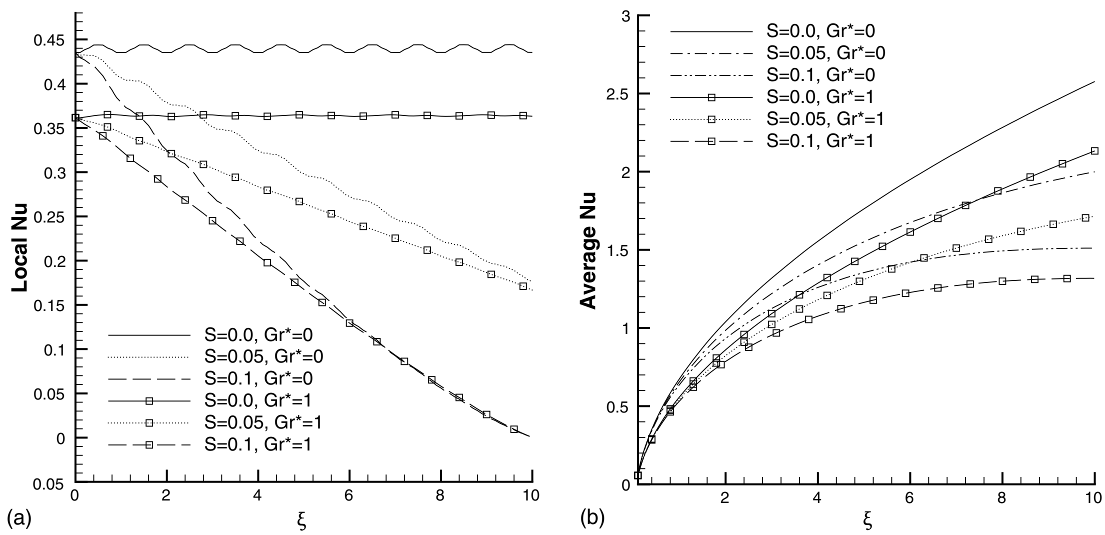

Fig. 2. (a) Local Nusselt number and (b) average Nusselt number plots when $S=0$, 0.05, 0.1, $a=0.2$, $\phi=0°$ under Darcian and non-Darcian assumptions.

to a gradual fall in the amplitude of the wavy pattern. At large S far away from the leading edge of vertical wavy wall local heat flux reduction gets monotonic. In the presence of inertial forces due to non-Darcian assumptions the fall in local heat fluxes is more or less monotonic with increasing ξ at all levels of thermal stratification.

In Fig. 2(b) average Nusselt number (Nu_m) plots along the wavy wall corresponding to the above two local heat flux plots are presented. At all levels of thermal stratification (S) along the lower portion of the wavy wall (i.e. when $\xi \prec 6$) there is an increase in average Nusselt number. Along the upper portion of the wavy wall, for large S (i.e. when $S \succ 0.05$) there is only a slow or marginal variation in average Nusselt number, which is a direct consequence of the fall in local heat fluxes (Nu_ξ) along the upper portion of wavy wall due to thermal stratification. The fall in local Nusselt number with increasing S has lead to the fall in average Nusselt number along the wavy wall under both Darcian and non-Darcian assumptions.

To analyse the near and far flow field and temperature distributions, streamlines and isotherms in the form of f contours and temperature contours are traced for the above mentioned parameter setting both for Darcian and non-Darcian cases. In Fig. 3(a), (c), (e) and (g) f

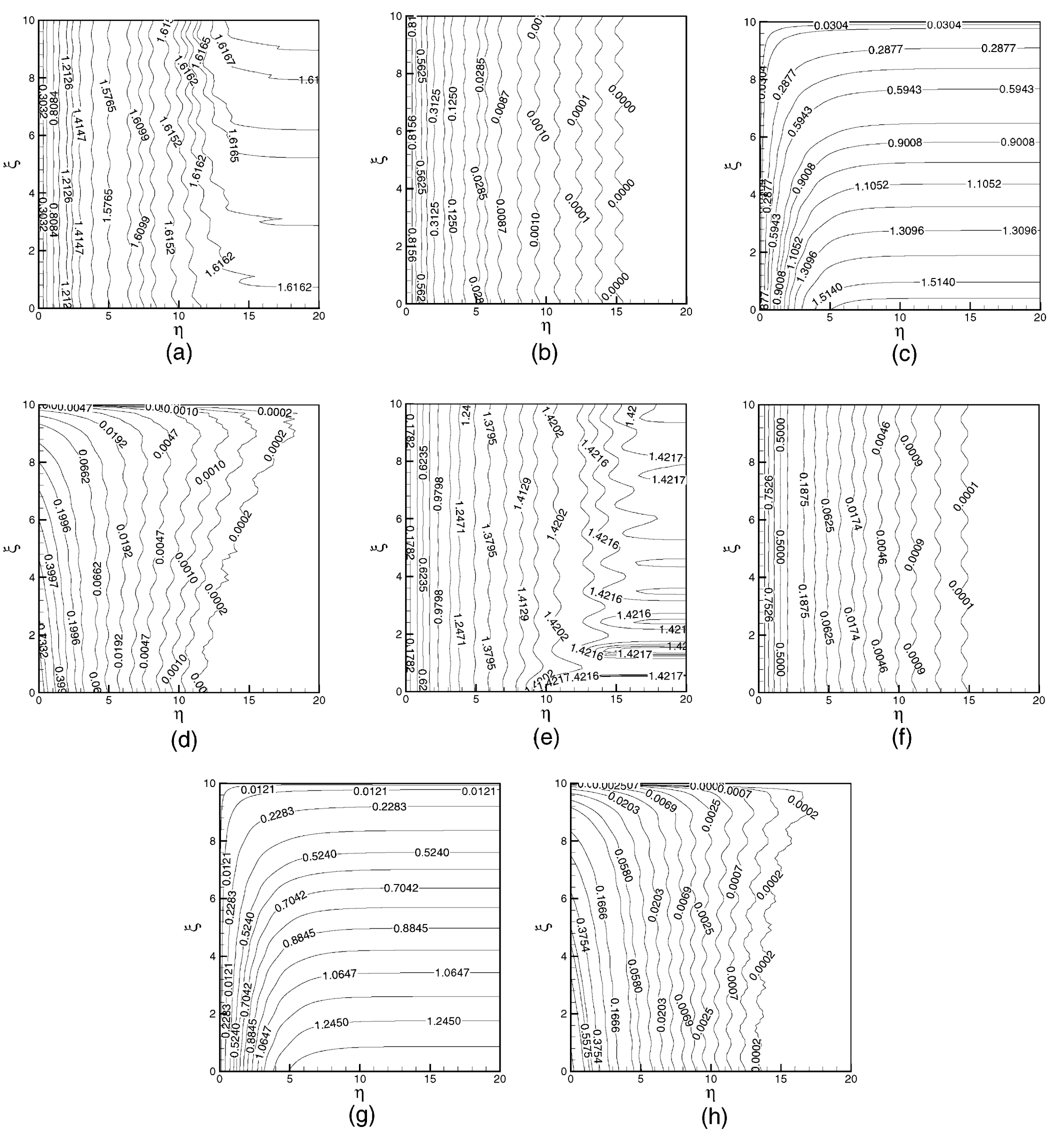

Fig. 3. (a) f contours with $S = 0$, (b) T contours with $S = 0$, (c) f contours with $S = 0.1$, (d) T contours with $S = 0.1$ when $Gr^* = 0$, $a = 0.2$, $\phi = 0°$ and corresponding f and T contours in (e–h) when $Gr^* = 1$, $a = 0.2$, $\phi = 0°$.

contours for Darcian and non-Darcian cases are presented when $S = 0,\ 0.1$, $a = 0.2$ and $\phi = 0°$. In the presence of thermal stratification f contours get horizontal with increasing η depicting a reduced flow situation. From the isotherms in Fig. 3(b), (d), (f) and (h) corresponding to Darcian and non-Darcian assumptions with $S = 0,\ 0.1$, one can note that in the presence of thermal stratification, heat transfer due to free convection into the porous medium tends to get zero with increasing η in tune with the no-flow situation as inferred from f contour pattern. Isotherms label values indicate the presence of thermal boundary layer adjacent to the wavy vertical wall. On comparing the isotherms corresponding to non-Darcian model (i.e. Fig. 3(h)) with those corresponding to Darcian model (i.e. Fig. 3(d)) one can visualize that the inertial forces enhance the free convection heat transfer range in the porous medium. The development of the wavy pattern in the temperature distribution with increasing η can be attributed to the transformation, given by Eq. (6), introduced in the η-direction to map the wavy surface to a flat surface.

3.2. Influence of amplitude-to-wavelength ratio (a) on free convection process

In Fig. 4(a) and (b) local Nusselt number (Nu_ξ) plots, in a thermally stable porous medium, along the wavy

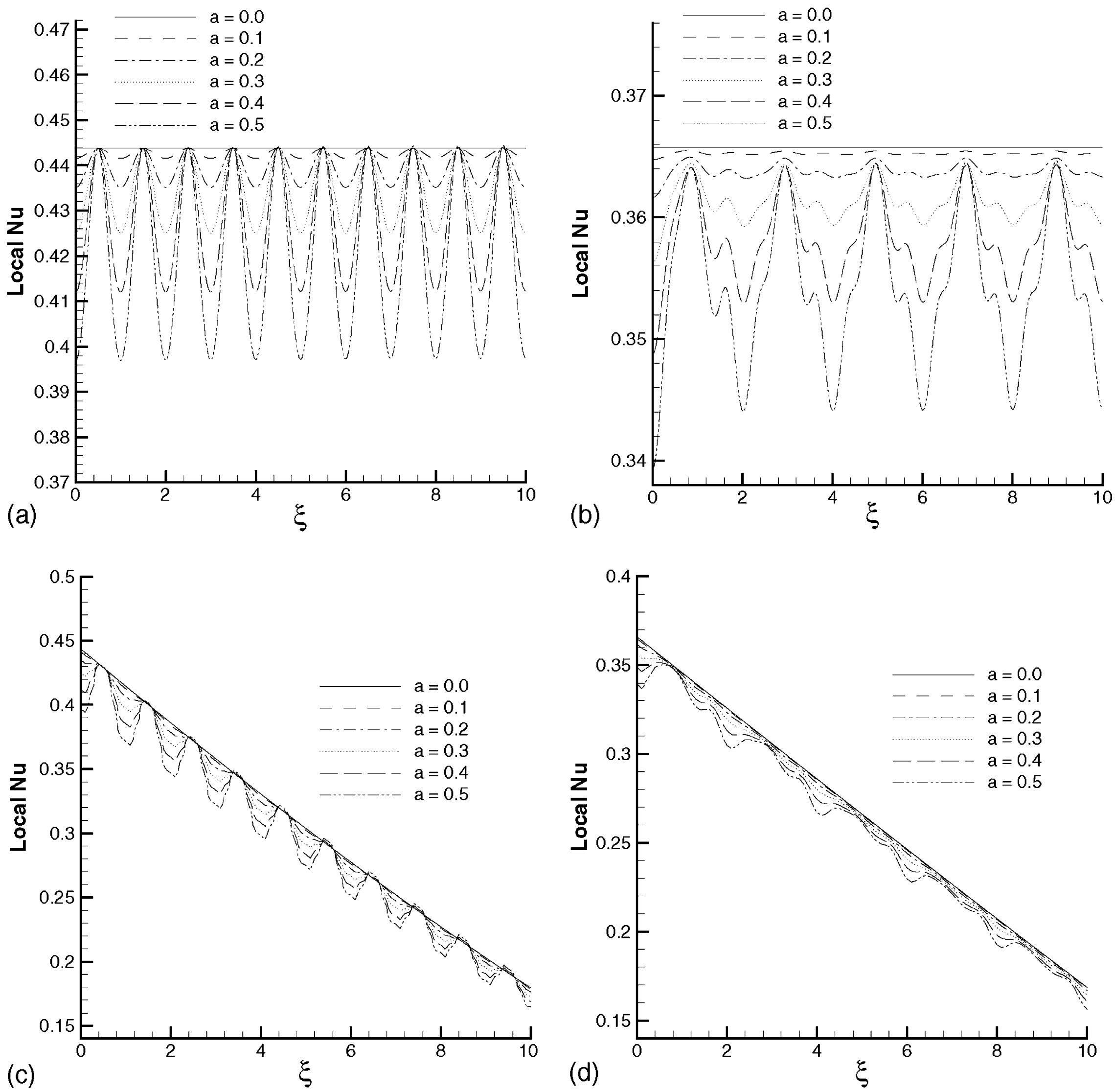

Fig. 4. Local Nusselt number plots for varying 'a' with $S = 0$, $\phi = 0°$ when (a) $Gr^* = 0$, (b) $Gr^* = 1$ and with $S = 0.1$, $\phi = 0°$ when (c) $Gr^* = 0$, (d) $Gr^* = 1$.

vertical surface are presented for $a = 0.0$, 0.1, 0.2, 0.3, 0.4, 0.5, $\phi = 0°$ when $Gr^* = 0.0$, 1.0 and $S = 0.0$. Both under the Darcian and non-Darcian assumptions, there is a periodic variation in local Nusselt number or local heat fluxes along the wavy wall. Here one can note that in the Darcian case the wavelength of the wavy local Nusselt number plots is exactly the half wavelength of the wavy surface whereas in the non-Darcian case it is same as that of the wavy surface. The nodes of the wavy surface are located at $\xi = 0$, 1, 2, 3, 4 etc., the troughs of the wavy surface are located at $\xi = 1.5$, 3.5 etc. and the crests are situated at $\xi = 0.5$, 2.5 etc. While the minimum local heat fluxes (or Nu_ξ) occur on the node of the wavy surface, the maximum local heat fluxes occur on the crests and troughs of the wavy surface. From Eqs. (7) and (8), when $Gr^* = 0$, the coefficient of highest order differential terms contains σ_ξ^2. Since σ is a periodic function with wavelength of 2ℓ, σ_ξ^2 is also periodic with wavelength ℓ. Therefore the solution of Eqs. (7) and (8) becomes periodical function with wavelength of ℓ. Consequently, the local Nusselt numbers, which are obtained from the solution of (7) and (8), also get periodic with wavelength of ℓ. From Fig. 4(a) and (b), one can notice that in the presence of inertial forces the local Nusselt number plot waves get complex but remain periodic. Here while the minimum heat fluxes occur at the nodes of the wavy surface at $\xi = 0$, 2, 4, 6 etc. leading to the crests, the maximum heat fluxes occur at the nodes of the wavy surface $\xi = 1$, 3, 5 etc. leading to the troughs. One can also find a local maximum and a local minimum in local heat fluxes wavy pattern between consecutive global minima and global maxima taken strictly in that order. Unlike the case when $Gr^* = 0$, the coefficient of highest order

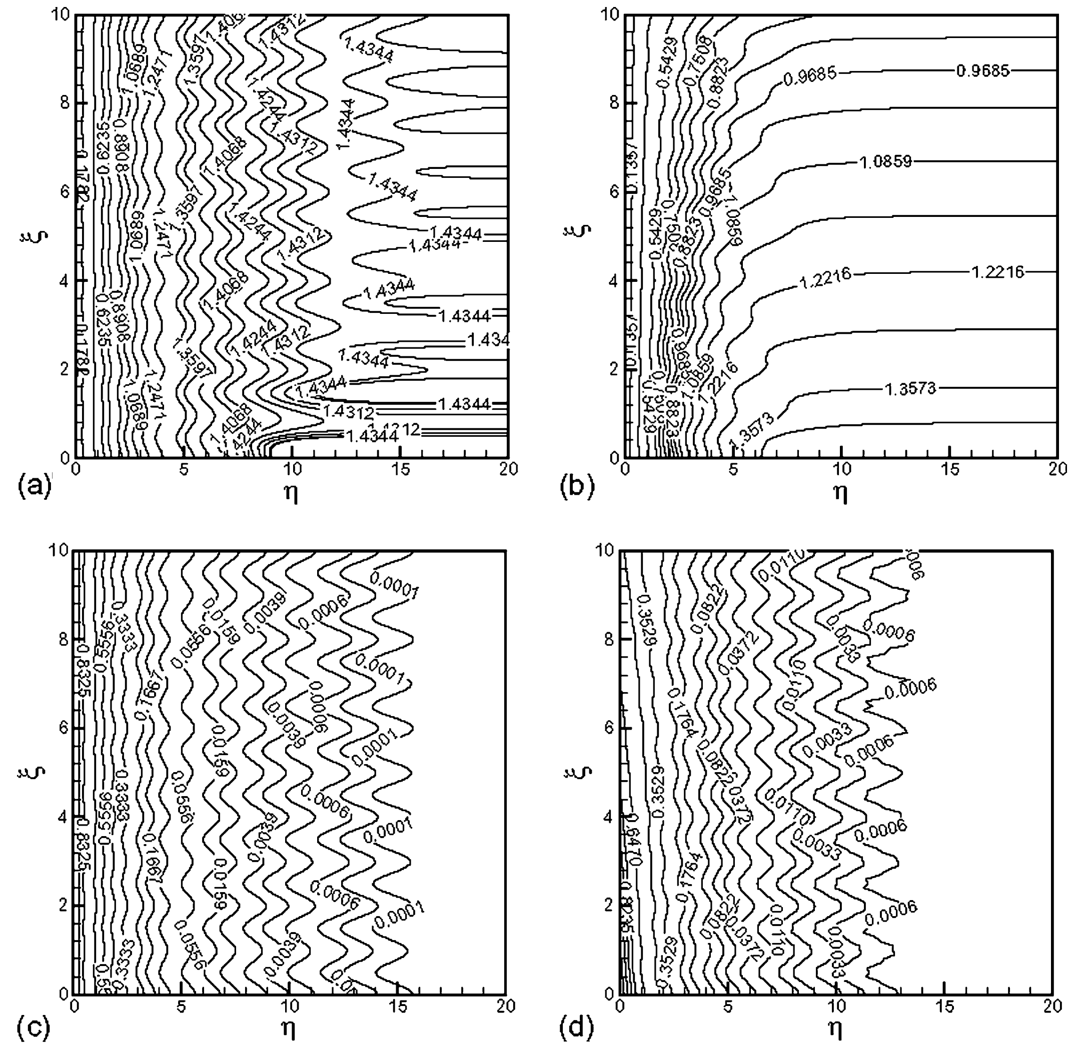

Fig. 5. f contours with $Gr^* = 1$, $\phi = 0°$, $a = 0.4$ when (a) $S = 0$, (b) $S = 0.05$ and corresponding T contours are in (c,d).

differential term contains σ_{ξ}^{3} when $Gr^{*} > 0$. σ_{ξ}^{3} is periodic with a complex wave pattern but with a period same as σ. This explains the complexities observed in the local Nusselt number plots in the presence of inertial forces. The fall in the magnitude of the local heat fluxes under non-Darcian assumptions is very much in agreement with the results corresponding to flat vertical wall as reported by Plumb and Huenefeld [20]. With increasing amplitude to wavelength ratio while the global maximum in local heat fluxes remains unaltered, the global minimum is seen to decrease owing to the reduction in the convection favoring buoyancy forces at nodal locations on the wavy surface corresponding to minimum heat fluxes.

Fig. 4(c) and (d) display the local Nusselt number pattern along the wavy surface in the presence of thermal stratification for various values of 'a'. Here too local heat fluxes depict a wavy pattern with features similar to those observed in thermally stable case. In addition, owing to the thermal stratification there is a drastic fall in the local heat fluxes as one move along the wavy wall. In the presence of inertial forces, like in the thermally stable porous medium case, the wavy pattern in the local heat flux variation along the wavy wall get complex with both global and local maxima and minima in the local heat flux plots.

The average Nusselt number plots corresponding to $Gr^{*} = 0$, 1, $S = 0$, 0.05, $\phi = 0°$ and $a = 0$, 0.1, 0.2, 0.3,

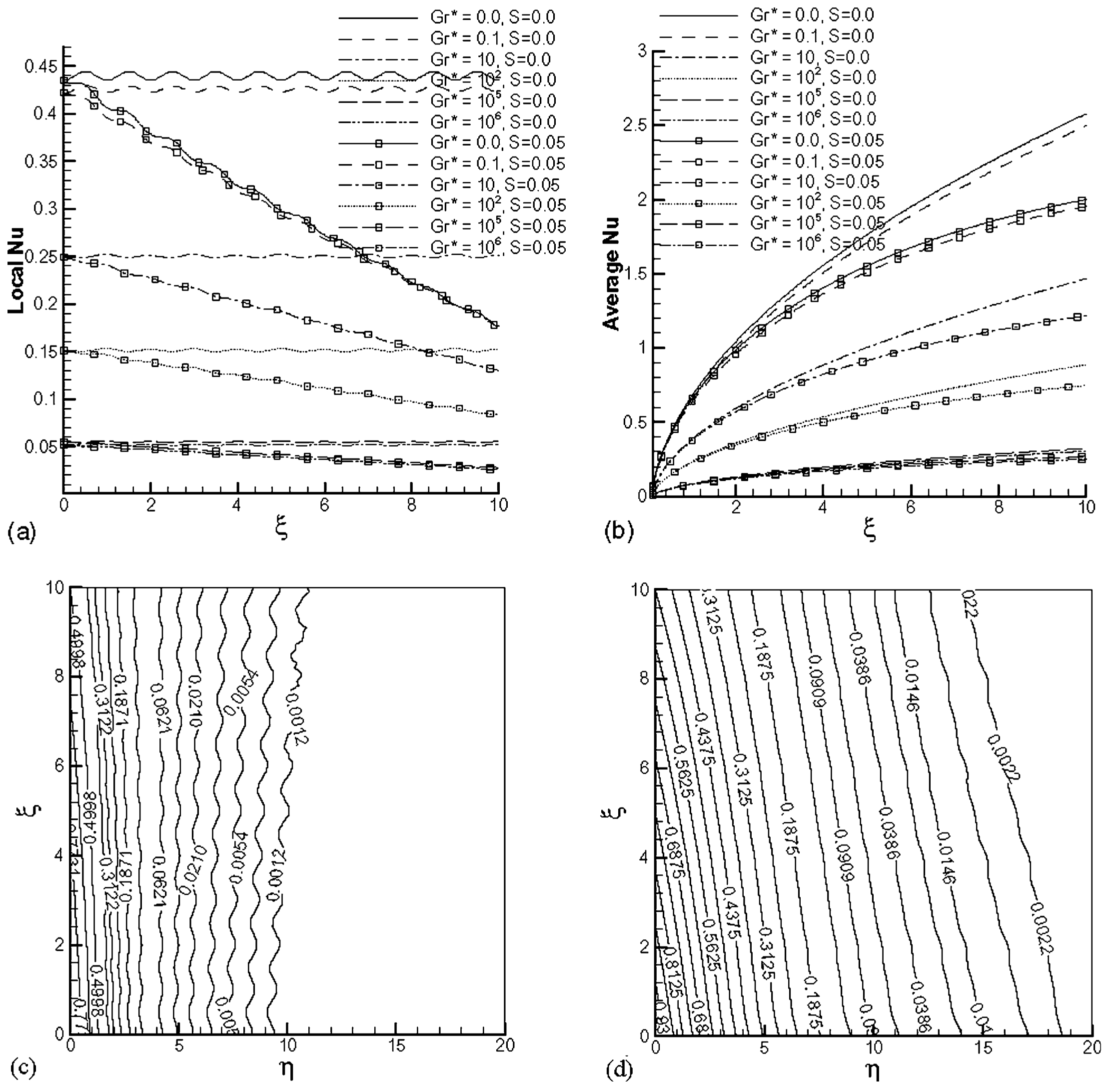

Fig. 6. (a) Local Nusselt number and (b) average Nusselt number when $a = 0.2$, $\phi = 0°$, $S = 0$, 0.05, $Gr^{*} = 0$, 0.1, 10, 10^2, 10^5, 10^6, T contours when (c) $Gr^{*} = 0$, (d) $Gr^{*} = 10^2$ and $S = 0.05$, $a = 0.2$, $\phi = 0°$.

0.4, 0.5 are also traced. Both in the presence and absence of inertial forces thermal stratification is seen to bring in a reduction in the average Nusselt number. Also average Nusselt number along the wavy surface decreases with increasing amplitude-to-wavelength ratio (a). This fall in the average Nusselt number is larger when $Gr^* = 0$.

Streamlines or f contours corresponding to $Gr^* = 0$, 1 both under thermally stable and stratified conditions are traced and analyzed for several values of 'a' and 'S'. In thermally stable porous medium flow induced by free convection tends to cover distances as large as $\eta \approx 15$ from wavy surface. Whereas in the case of thermally stratified porous medium free convection induced flow gets marginalized at distances as small as $\eta \approx 7$ from the wavy surface. In both the cases these distances are found to be sensitive to amplitude-to-wavelength ratio.

In Fig. 5(a) and (b) f contours corresponding to $a = 0.4$, $Gr^* = 1$ when $S = 0$, 0.05 are presented. Isotherms corresponding to $a = 0.4$, $Gr^* = 1$ when $S = 0$, 0.05 are shown in Fig. 5(c) and (d). Clearly in the presence of thermal stratification the region of porous medium under influence of free convection heat transfer from wavy wall is reduced. Also the extent of the region under influence of the convection process is seen to be sensitive to the wavelength-to-amplitude ratio. Presence of thermal boundary layer adjacent to wavy wall can be inferred from the isotherm label values.

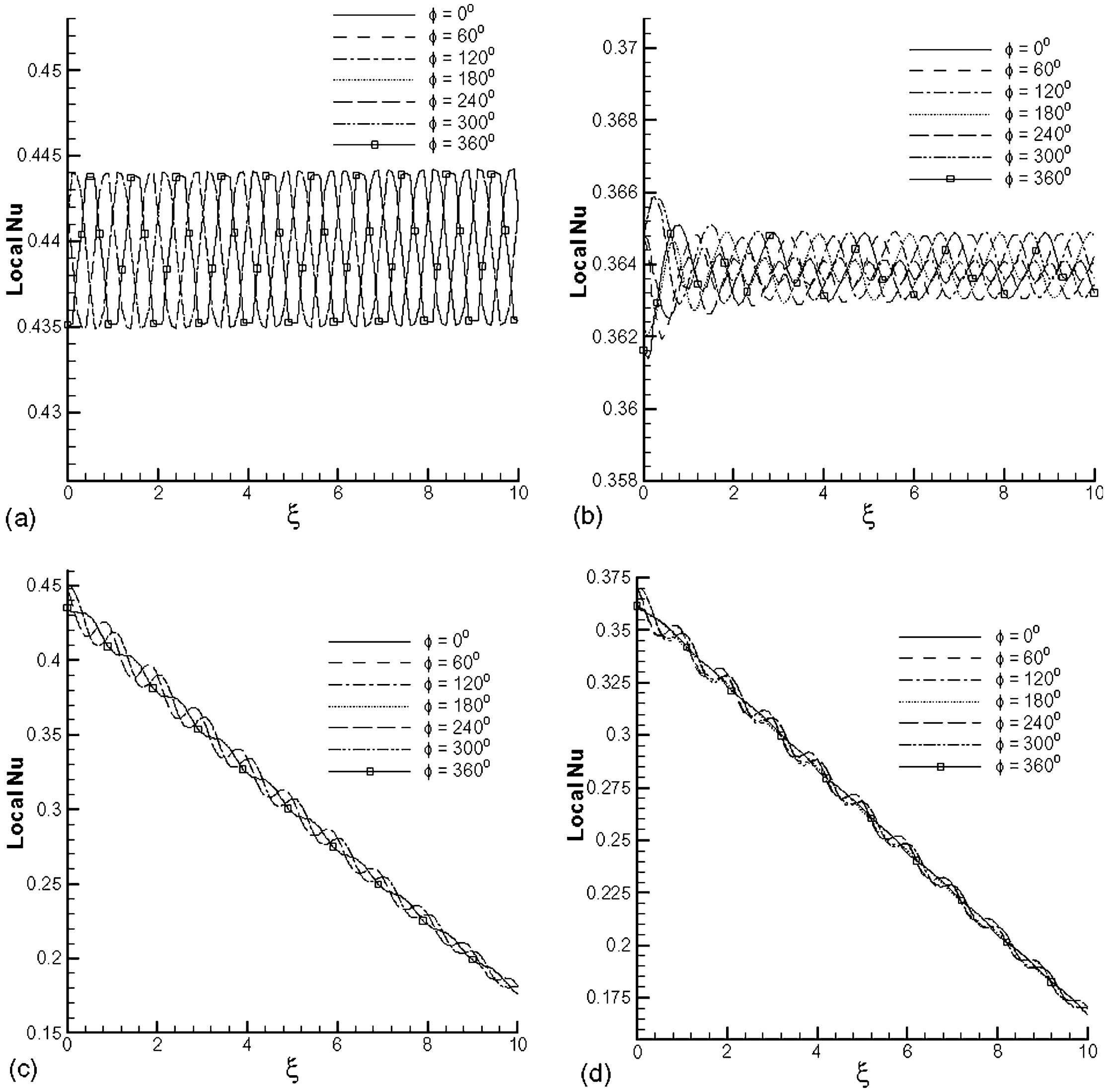

Fig. 7. Phase (ϕ) variation effect on local Nusselt number when (a) $Gr^* = 0$, $S = 0$, $a = 0.2$, (b) $Gr^* = 1$, $S = 0$, $a = 0.2$, (c) $Gr^* = 0$, $S = 0.05$, $a = 0.2$ and (d) $Gr^* = 1$, $S = 0.05$, $a = 0.2$.

3.3. Influence of inertial forces due to Forchheimer terms on free convection process

Plots in Fig. 6(a) depict the effect of inertial forces on free convection process when $S = 0$, 0.05 for $Gr^* = 0$, 0.1, 10, 10^2, 10^5, 10^6 setting $a = 0.2$, $\phi = 0°$. Both in thermally stable and thermally stratified porous media increasing Gr^* drastically reduces the local heat transfer from the wavy vertical wall all along the wavy surface. The wavy pattern in local Nusselt number variation is a direct consequence of the influence of undulations on vertical surface. Plots also depict that at large values of Gr^* extent of the fall in local Nusselt number along the wavy wall gets significantly reduced in the presence of thermal stratification. On increasing Gr^* both when $S = 0$ and $S = 0.05$, magnitude of local Nusselt numbers seem to reach a saturation level at which any further increase in Gr^* has only a marginal influence on local heat fluxes. Average Nusselt number plots in Fig. 6(b) corresponding to $S = 0$, 0.05 when $a = 0.2$, $\phi = 0°$ and varying Gr^* show a drastic reduction in average Nusselt numbers along the wavy surface when Gr^* is increased. Isotherm plots depict a marked variation in the temperature distribution in thermally stable porous medium with increasing Gr^*. The isotherm label values near the vertical wavy surface suggest that the thermal boundary layer which is quite sharp when $Gr^* = 0$ gets blunt with increasing Gr^*. This is very much in agreement with the loss of the local heat fluxes along the wavy wall on increasing Gr^*. However, with increasing Gr^* extent of the spread of free convection heat from wavy surface into both thermally stable and thermally stratified porous medium is increased. Isotherms corresponding to $Gr^* = 0$, 10^2 when $S = 0.05$ and $a = 0.2$ are shown in Fig. 6(c) and (d).

3.4. Influence of wave phase (ϕ) on free convection process

In Fig. 7(a) and (b) variation of local Nusselt number for $a = 0.2$, when $S = 0$ and $Gr^* = 0$, 1 with phase (ϕ) are presented. The change in the spatial location of maxima and minima in the wavy pattern of local heat fluxes with the change in the wave phase is due to shift in the spatial location of wave nodes, crests and troughs of the wavy vertical surface with the changing phase. In Fig. 7(c) and (d) local Nusselt number for $a = 0.2$ when $S = 0.05$ and $Gr^* = 0{,}1$ with varying ϕ are presented. Here, amplitude of the wavy local heat flux pattern is relatively larger when $\phi = 60°$, 120°, 240° and 300° in comparison to the case when $\phi = 0°$, 180°, 360°. At all values of ϕ, while the presence of inertial forces lead to continuous reduction in magnitude of local heat flux all along the vertical wavy surface, the presence of thermal stratification leads to a nearly monotonic fall in local heat flux curves along the vertical wall.

4. Conclusions

In this study free convection induced by a vertical wavy surface in a thermally stratified porous medium is analysed and compared with the free convection process in a thermally stable porous medium both under Darcian and non-Darcian assumptions. Using a non-similarity transformation the partial differential equations governing the natural convection process are simplified. These simplified partial differential equations are solved numerically in finite difference frame work following the Keller Box approach. Numerical simulations are carried out for various parameters governing the convection process.

- In the presence of thermal stratification local Nusselt number falls along the wavy wall as one moves away from leading edge.
- For large values of thermal stratification fall in local Nusselt number along the wavy wall gets nearly monotonic.
- Inertial forces enhance the extent of spread of natural convective heat transfer into porous medium both in thermally stable and thermally stratified porous medium.
- In Darcian case local Nusselt number variation is periodic with a wavelength 'ℓ' which is half of the wavelength of wavy vertical wall. In the Darcian case maximum values of local Nusselt number occur at the locations corresponding to crests or troughs of wavy wall while the minimum local heat fluxes always occur at the nodes of wavy wall. In the presence of inertial forces local Nusselt number variation remains periodic with wavelength 2ℓ but gets complex with a local maxima and a minima in every period apart from having a global maxima and a minima.
- Sharp thermal boundary layers are noticed adjacent to wavy wall where Gr^* is small.
- Effectively on increasing either thermal stratification level or wavelength-to-amplitude ratio or modified Grashof number lead to a fall in local heat fluxes.

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